**Implementation of the IFD method in pricing a European Call Option on a Pure Discount Bond in the CIR framework.**

Consider the CIR model of short-term rate: .

The goal is to compute the price of the European Call option on a pure discount bond, which matures at time *S* and pays *FV* at maturity. The option’s expiration is at time T , and the strike price of the option is *K*.

Under the risk-neutral measure, the price of the call option is given by:

where is the price of the pure discount bond at time T, maturing at time S and paying FV at maturity.

Using the Feyman-Kac Theorem, we can transform the problem into another one in which we solve the following PDE:

With the terminal condition, where is the price of the pure discount bond at time T, maturing at time S and paying FV at maturity.

While there is a closed-form solution to the above-posed problem, below we provide the details of numerically solving the PDE, by using the Implicit Finite Difference (IFD) method.

Consider a grid of time (t) and rate (r) as follows:

Take a uniform partition of the time interval [0,T]. Divide the time-interval [0,T] by equal parts:

, where , where

Take a truncated range of *r* as follows: takes on these values . That is, , for any .

The boundary conditions/values of option prices can be used as follows:

1. for (this is the payoff condition at option’s expiration)
2. , for (since for very large values of r, the call option is deep out of the money, and the call value is worthless
3. , for (

The discretized version of this PDE (using the Implicit FD method) can be written as follows:

By combining the similar terms, we can rewrite the above scheme as:

where

for .

Also, we have the boundary conditions for *C*: and for .

Putting all equations together, we will have the following system of equations in a matrix form:

where

; ;

The goal is to find .

The matrix equations can efficiently be solved, starting at and moving backwards in time

**Remarks:**

1. We did not include the extreme values of *C* in the matrix equations above. That is, the values of and , which correspond to option values for “very large” and “very small” bond prices, respectively, were not included in the matrix equation above.
2. One obtains a “very large” (or small) bond price when the rate is 0 (or ).
3. For call options, and for we can use as this is the case when *r=0* and the call option is deep in-the-money. Also, , for . This corresponds to the case when , when the underlying bond price is very low, so the option is close-to-being worthless.